

**MATH 324 Real Analysis I**

(4 Lectures, 0 Laboratory – 4 Credits)

**Prerequisite:** MATH 101 and MATH 112

**Course Catalog Description:**

This course gives students a thorough understanding of essential concepts in analysis such as real numbers, limits, continuity, and convergence of sequences and series. The course also covers a rigorous definition of derivative and construction of the Riemann integral and their properties including the Fundamental Theorem of Calculus. Students are required to read and write proofs using a precise knowledge of definitions and theorems.

**Textbook:**

*Analysis with an Introduction to Proof* by Steven R. Lay, Publisher: Pearson, 5<sup>th</sup> Edition, 2005, ISBN: 9780321747471.

**Reference Materials:**

- *Undergraduate Analysis* by Serge Lang, Publisher: Springer Verlag, 2<sup>nd</sup> Edition, 2005, ISBN: 9788184896282.
- *Elementary Analysis* by Kenneth A. Ross, Publisher: Springer, 2<sup>nd</sup> Edition, 2013, ISBN: 9781461462705.

**Course Topics:**

1. Introduction: review of tautologies, contradictions and equivalence, open sentences and quantifiers; methods of proof in mathematics: direct method, contrapositive method, contradiction method (3 lectures)
2. Sets and functions: indexed families of sets and their properties, image and inverse image of functions and their properties (5 lectures)
3. Real number systems: countability of  $\mathbb{R}$  and  $\mathbb{Q}$ , order properties of  $\mathbb{Q}$  and its order incompleteness, construction of  $\mathbb{R}$  from  $\mathbb{Q}$  using Dedekind cuts axioms (6 lectures)
4. Order completeness of  $\mathbb{R}$ : least upper bound property and equivalent conditions including the nested interval property, bounded sets, and their properties, supremum and infimum of sets, Bolzano-Weierstrass theorem (7 lectures)
5. Topological properties of  $\mathbb{R}$ : interior points, limit points, isolated points, boundary points, frontier points, open sets, closed sets, perfect sets, compact sets, and connectedness (6 lectures)
6. Sequences: definition of convergent sequence, bounded sequences, monotone sequences and their convergence, limsup and liminf and convergence criterion, subsequences, Cauchy sequences and their convergence criterion (8 lectures)
7. Limits and continuity: epsilon-delta definition of limit, basic properties of continuous functions, uniform continuity, bounded functions, continuous functions on compact sets, Intermediate Value Theorem (10 lectures)
8. Differentiation: derivatives, the Mean Value theorem, L'Hospital's Rule, Taylor's theorem (5 lectures)
9. Riemann integral: definition, properties, the Fundamental Theorem of Calculus (6 lectures)
10. Series: series of numbers, definition of convergent series, geometric series, the comparison test, integral test with proof, ratio test, absolute and conditional convergence, alternating series and Leibniz test (4 lectures)

**Structure and Learning Methodologies:**

Lectures will be either 4 x 50 minutes per week or 2 x 75 and 1 x 50 minutes per week. Lectures will be presented using the whiteboard, and may be complemented with handouts and/or PowerPoint slides.

**Assessment:**

All course learning outcomes are assessed using the following assessment tools.

Coursework (Quizzes and assignments)	40%
Semester Examination	25%
Final Examination	35%

**Contribution to Applied Mathematics & Statistics Students Outcomes (AMS-SOs):**

a	b	c	d	e	f	g	h	i	j	k	l
H	---	H	---	---	---	---	---	---	---	---	---

H – High      M – Medium      L – Low

**Course Learning Outcomes & AMS-SOs:**

No	Course Learning Outcomes	AMS-SOs
1	Identify and prove topological properties of sets of real numbers.	a, c
2	Determine and prove the main properties of sequences.	a, c
3	Prove the continuity of a function.	a, c
4	Prove the convergence/divergence of an infinite series.	a, c
5	Prove classical theorems in real analysis.	a, c